Student Workbook

Unit Review — Trig Graphs & Inverses

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How do the graphs of the six trigonometric functions and their inverses connect the unit circle to real-world periodic behavior?

Lesson Overview

Unit 6 covers three major topics. In 6.1 you graphed sine and cosine, identifying amplitude, period, phase shift, and vertical shift from $y = A\cdot\sin(Bx-C)+D$. In 6.2 you extended this to tangent, cotangent, secant, and cosecant, focusing on vertical asymptotes and period differences. In 6.3 you defined the three inverse trig functions, their restricted domains, and how to compose trig with inverse trig. This review consolidates all three topics.

$$y = A\cdot\sin(Bx - C) + D$$

Amplitude = $|A|$ Period = $2\pi/|B|$ Phase shift = C/B Midline: $y = D$

6.1/6.2	Period of tan/cot = $\pi/|B|$; asymptotes of tan/sec at $\pm\pi/2+n\pi$, of cot/csc at $n\pi$
6.3	arcsin: $[-1,1]\to[-\pi/2,\pi/2]$; arccos: $[-1,1]\to[0,\pi]$; arctan: $(-\infty,\infty)\to(-\pi/2,\pi/2)$
Compose	For sin/cos/tan of an inverse trig expression, draw a right triangle

Worked Examples

EXAMPLE 1

Identify the amplitude, period, phase shift, and vertical shift of $y = 3\sin(2x-\pi)-1$. Then state the range.

- Standard form: $A=3$, $B=2$, $C=\pi$, $D=-1$
- Amplitude = $|3| = 3$. Period = $2\pi/2 = \pi$. Phase shift = $\pi/2 = \pi/2$ to the right. Vertical shift = -1
- Range = midline ± amplitude = $[-1-3, -1+3]$

Answer: Amplitude 3, period π, phase shift $\pi/2$ right, vertical shift -1, range $[-4, 2]$

EXAMPLE 2

Sketch one full period of $y = 2\sin(2x-\pi/2)+1$ and label the five key points.

- $A=2$, $B=2$, $C=\pi/2$, $D=1$: amplitude=2, period=π, phase shift=$\pi/4$ right, midline $y=1$
- Key x-values start at $\pi/4$, add quarter-periods ($\pi/4$): $\pi/4$, $\pi/2$, $3\pi/4$, π, $5\pi/4$
- Key y-values follow the sine pattern (start,max,mid,min,end): 1, 3, 1, -1, 1

Answer: Key points: $(\pi/4,1)$, $(\pi/2,3)$, $(3\pi/4,1)$, $(\pi,-1)$, $(5\pi/4,1)$

EXAMPLE 3

State the period and vertical asymptotes of $y = \tan(3x-\pi/4)$.

- Period = $\pi/|B| = \pi/3$
- Asymptotes where argument = $\pi/2+n\pi$: $3x-\pi/4 = \pi/2+n\pi \to 3x = 3\pi/4+n\pi$

Answer: Period = $\pi/3$; asymptotes at $x = \pi/4 + n\pi/3$, for all integers n

EXAMPLE 4

Evaluate: (a) $\arcsin(-\sqrt{2}/2)$ (b) $\arccos(0)$ (c) $\arctan(-\sqrt{3})$

- (a) Need θ in $[-\pi/2,\pi/2]$ with $\sin\theta=-\sqrt{2}/2$: $\theta=-\pi/4$
- (b) Need θ in $[0,\pi]$ with $\cos\theta=0$: $\theta=\pi/2$
- (c) Need θ in $(-\pi/2,\pi/2)$ with $\tan\theta=-\sqrt{3}$: $\theta=-\pi/3$

Answer: (a) $-\pi/4$ (b) $\pi/2$ (c) $-\pi/3$

EXAMPLE 5

Simplify: (a) $\sin(\arcsin(3/5))$ (b) $\cos(\arcsin(5/13))$ (c) $\tan(\arccos(x))$ for $x\in(0,1)$

- (a) $\sin(\arcsin(u))=u$ directly: 3/5
- (b) $\theta=\arcsin(5/13)$: opposite=5, hyp=13, adjacent=$\sqrt{169-25}=12 \to \cos\theta=12/13$
- (c) $\theta=\arccos(x)$: $\cos\theta=x$, $\sin\theta=\sqrt{1-x^2} \to \tan\theta = \sqrt{1-x^2}/x$

Answer: (a) 3/5 (b) 12/13 (c) $\sqrt{1-x^2}/x$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Find the amplitude, period, phase shift, and vertical shift of $y = -4\cos(\pi x+\pi/2)+2$.

Hint: Write as $y=A\cdot\cos(Bx-C)+D$. Factor B out of the argument: $\pi(x+1/2)$, so $C/B=-1/2$ (shift left).

2 Give the period and two consecutive asymptote equations for $y = \cot(x/2)$.

Hint: Period of $\cot(Bx)=\pi/|B|$. Asymptotes of cot occur where the argument = $n\pi$.

3 Evaluate $\arctan(1) + \arccos(-1)$.

Hint: $\arctan(1)=\pi/4$ since $\tan(\pi/4)=1$. $\arccos(-1)=\pi$ since $\cos(\pi)=-1$.

4 Write an equation $y=A\cdot\sin(Bx)$ for a sine wave with amplitude 5 and period 4π.

Hint: Amplitude=$|A|$=5. Period=$2\pi/B=4\pi$, so solve for B.

5 Simplify sin(arccos(-4/5)).

Hint: Let θ=arccos(-4/5), θ∈[0,π]. Draw a right triangle with adjacent=-4, hypotenuse=5, and find the opposite side.

Key Vocabulary

Amplitude	|A| — half the distance between a sine/cosine wave's maximum and minimum values.
Period	The horizontal length of one complete cycle: 2π/|B| for sin/cos, π/|B| for tan/cot.
Phase Shift	The horizontal shift of a trig graph, found as C/B once B is factored out of the argument.
Vertical Shift (Midline)	D — the horizontal line y=D that a sine or cosine graph oscillates around.
Vertical Asymptote	A line the graph approaches but never crosses — occurs for tan, cot, sec, and csc.
Inverse Trig Function	arcsin, arccos, arctan — functions that undo sin, cos, tan on a restricted domain.
Restricted Domain	The limited input interval used to make a trig function one-to-one so it has an inverse.
General Form	y = A·sin(Bx-C)+D (or cosine) — identifies amplitude, period, phase shift, and vertical shift at once.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **What is the period of y = sin(3x)?** Multiple Choice

A π/3
B 2π/3
C 3π
D 6π

2 **The graph of y = tan x has vertical asymptotes at x =** Multiple Choice

A nπ
B π/2 + nπ
C 2nπ
D π/4 + nπ

3 **What is arcsin(1)?** Multiple Choice

A 0
B π/6
C π/2
D π

4 **For y = 2cos(x) - 3, what is the range?** Multiple Choice

A [-3, 3]
B [-5, -1]
C [-2, 2]
D [-1, 5]

5 **Which function has period π (not 2π)?** Multiple Choice

A y = sin x
B y = cos x
C y = tan x
D y = csc x

Common Mistakes to Avoid

✗ Period = 2π · B (multiplying instead of dividing).	✓ Period = 2π/|B|. A larger B compresses the graph, making the period shorter.
✗ Phase shift = C (reading the constant directly).	✓ Phase shift = C/B. Always divide by B after factoring the argument.
✗ arcsin(sin(3π/2)) = 3π/2.	✓ arcsin outputs values in [-π/2,π/2] only. sin(3π/2)=-1, so arcsin(-1)=-π/2.
✗ Assuming the asymptotes of sec x are the same as tan x.	✓ sec x=1/cos x has asymptotes where cos x=0 (x=π/2+nπ) — same as tan, but csc x=1/sin x has asymptotes at x=nπ.

Math Tips

★ Always factor B out of the argument before reading phase shift: $A \cdot \sin(B(x-h))+D$ -> phase shift = h.

★ The five key points of one sine/cosine period are evenly spaced at quarter-period intervals.

★ For inverse trig compositions, draw a right triangle — faster and less error-prone than memorizing formulas.

★ sec and csc are reciprocals of cos and sin respectively; their graphs hug the parent curve at its peaks and valleys.

★ Remember: arcsin and arctan output angles in $(-\pi/2, \pi/2)$; arccos outputs angles in $[0, \pi]$.